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NONLINEAR FEEDBACK CONTROL FOR HIGH ALPHA FLIGHT

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PROBLEM UNDER INVESTIGATION

The central problem under investigation is that of constructing via optimal control theory analysis time-optimal maneuvers to reverse directions of flight. Consider a high alpha fighter aircraft flying North at 0.6 Mach under trim conditions. What is the time-optimal maneuver to reverse direction and end up flying South at 0.6 Mach under trim conditions and with the same final energy as initially? This is an unsolved six degree of freedom (6 DOF) high alpha flight path optimization problem. An aircraft's entire agility is scrutinized in performing this single maneuver in minimum time. Almost all flight optimization work using optimal control theory analysis is based on point mass equations of motions. The neglect of moments equations assume: (1) angular rate contributions to the forces are small, (2) unsteady effects are small, (3) certain states can change instantly from any value to any other value and therefore can be treated as control variables and (4) thrust vectoring needs no counter-balancing aerodynamic moment. Since poststall benefits are at low speeds and high alpha, in the presence of unsteady flow and with thrust vectoring it is becoming more difficult to justify the neglect of moment equations in optimal control analysis of poststall aircraft flight. Our flight optimization work objective is to solve the above problem using optimal control theory analysis based on 6 DOF equations of high alpha flight. Our preliminary investigation attacks this problem by first analyzing some basic maneuvers such as half-loop, pitch-ups and level turns. The results of this preliminary work is presented below in Sections 3.1-3.3 and in references [45-47,53] and in some of the work leading up to that contained in [48].

In general we are interested in the optimal control problem of synthesizing an aircraft's agility into time-optimal maneuvers. Of particular interest are the shapes and forms in space of optimal high alpha flight trajectories, confirming classical tactics and strategies, establishing new ones and yielding any improvements stemming from high alpha flight, thrust vectoring, etc. The optimal control solutions for minimum time maneuvers are to be based on both moment and force equations.

ABSTRACT

Analytical aerodynamic models are derived from a high alpha 6 DOF wind-tunnel model. One detail model requires some interpolation between nonlinear functions of alpha. One analytical model requires no interpolation and as such is a completely continuous model. Flight path optimization is conducted on the basic maneuvers: Half-loop, 90 degree pitch-up and level turn. The optimal control analysis uses the derived analytical models in the equations of motion and is based on both moment and force equations. The maximum principle solution for the half-loop is a poststall trajectory performing the half-loop in 13.6 seconds. We found that the agility induced by thrust vectoring capability provided a minimum effect on reducing the maneuver time. Without thrust vectoring we

found that the pitch-up to 74 degrees alpha took 1.7 seconds and that there was an energy barrier beyond the alpha of 74 degrees. The additional 16 degrees required over 7 seconds of pitch-up time for a total of 8.7 seconds. On the other hand, we found using thrust vectoring control that the analytical model could be pitched-up to 90 degrees alpha in only 1.5 seconds by using a maximum thrust vectoring angle of 20 degrees. The change in altitude is less than 100 feet. Consequently by means of thrust vectoring control the 90 degrees pitch-up maneuver can be executed in a small space over a short time interval. The agility capability of thrust vectoring is quite beneficial for pitch-up maneuvers. The level turn results are based currently on only outer layer solutions of singular perturbation. Poststall solutions provide high turn rates but generate higher losses of energy than that of classical sustained solutions. The results of work produced on this grant are contained in the following publications [44-47, 53] and in some of the work leading up to that in [48] referenced at the end of this report.

REPORT OUTLINE

The role of supermaneuverability in winning combat engagements is briefly discussed in Section 1.1. An aircraft's capability to be supermaneuverable is related to its ability to achieve high levels of agility. A review of "what is agility?" is given in Section 1.2. The problem of synthesizing agility for optimal maneuvering is highlighted in Section 1.3. Previous work on flight path optimization of high alpha flight using optimal control theory analysis is presented in Section 1.4. That work is based on point mass equations of motions. The previous work of the Principal Investigator that is related to flight path optimization based on moment equations as well as force equations is described in Section 1.5. The specific objectives of the research work of this grant are briefly stated in Section 1.6. One main objective is to derive an analytical aerodynamic model from wind-tunnel data of a high alpha fighter aircraft. The other is to construct time-optimal maneuvers for the high alpha model.

Several levels of analytical aerodynamic models are derived from wind-tunnel data for subsonic high alpha flight. Such models are very helpful in obtaining computerized results when optimal control theory is applied to the nonlinear equations of motion. These models are described in Sections 2.1-2.3. The most detailed analytical model is described in Section 2.1. It requires some interpolation between nonlinear functions of alpha. A simpler analytical model for the longitudinal mode is described in Section 2.2. A simpler lateral model is described in Section 2.3. These latter two require little or no interpolation.

The derived analytical models are used in optimal control analysis to generate maximum principle solutions of several basic maneuvers. The half-loop results are described in Section 3.1. The results of minimizing the time to perform the 90 degree pitch-up maneuver with and without thrust vectoring are described in Section 3.2. Level turns results are presented in Section 3.3.

1. Introduction and Background

1.1 Background on Supermaneuverability

The future design of combat aircraft is being driven in part by the philosophy that the bottom line of air combat is not to preserve or to maintain energy but to survive and to win engagements, Herbst [1, 2]. In combat against "all-aspect" air-to-air missiles the aircraft pointing first in the direction of his target will survive. Consequently, maneuvering means are sought which improve the offensive and/or defensive capability in a combat engagement and improve the exchange ratio. The ability to maneuver a combat plane and greatly improve its performance has been called by some authors "supermaneuverability". Supermaneuvers utilize post-stall maneuvers, [1-4], which vastly improve point/shoot capability through controlled pointing of the aircraft nose. Herbst, [2], refers to supermaneuverability as maneuvers with segments of flight in which maximum lift angle of attack is exceeded purposely and in a controlled manner and with others segments of flight which deviate from coordinated maneuvers at zero sideslip angle. More recently, Herbst, [3], defines a supermaneuver as a coordinated maneuver beyond stall limits with sideslips as close to zero as possible. In the context of increased combat performance for future fighter aircraft, Lang and Francis [4] use the term "supermaneuverability" to connote "very high levels of agility and maneuverability available throughout an extended flight envelope. This includes, for example, the capabilities (i) to rapidly accelerate or decelerate, (ii) to turn tightly and quickly, (iii) to change maneuver conditions rapidly such as the turning and longitudinal acceleration forces through rolling, pitching, aerodynamic flow management and thrust vector control, (iv) to obtain rapid fire control solutions and weapons delivery and (v) to rapidly disengage and move on to the next target. Obviously, the quality of a supermaneuver is a function of the levels of agility in the flight path. What is agility? We answer this by reviewing briefly the definitions as given by various authors.

1.1 Background on Agility

Lang and Francis [4] define "agility" as "the ability of an aircraft to move from one maneuver condition to another at a rapid rate". McAtee [5] defines agility as "the ability to point the aircraft at the enemy quickly, continue high turn rates through low energy loss during maximum maneuvers and accelerate quickly." McAtee categorizes the characteristics of agility under the terms "maneuverability" and "controllability" in which maneuverability refers to the ability to change the flight path vector and controllability refers to the ability to change the aircraft attitude and thrust. Lawless [6] defines agility as "the ability to change maneuver state quickly with precision and control." Lawless considered several types of agility measures: (1) load factor - the ability to reach a desired g-level with quickness and a certain amount of precision, (2) lateral turn - the ability to turn 180 degrees in the absolutely shortest time possible and (3) loaded roll - the ability to roll as quickly as possible to a desired bank angle, maintaining a load factor throughout the roll. In [7] Dorn advocates a pointing-vector agility measure and a velocity-vector agility measure. Dorn presents a 2x2 matrix in which he distinguishes between maneuverability and agility and between energy and angles. He defines energy-maneuverability as the ability to change energy state (climb and/or accelerate) for the purpose of creating a relative energy advantage. Energy-agility is defined as the ability to minimize the time-energy penalty while directly seeking a position advantage. Angles-maneuverability is defined as the ability to change relative position (separation and/or orientation) for the purpose of creating a relative position/orientation advantage. Finally, Dorn defines angles-agility as the ability to rapidly and efficiently convert a given energy into position advantage (to meet firing parameters). Baker [8] defines agility simply as "the rate at which an aircraft can change its state". He defines maneuverability as "the ability of an aircraft to execute a particular state

change or sequence of state changes." Furthermore, Baker defines controllability as "the ability of an aircraft to achieve a desired transient behavior in relation to its state changes (e.g., settling time, overshoot, tracking error, steady-state error)." Kalviste in [9-10] considers the agility parameters of (1) point-and-shoot and (2) roll reversal. Riley and Drajeste [11-13] consider a torsional agility metric and define it as "the ratio of the aircraft's turn rate to the time to roll and capture a 90 degree bank angle change from -45 degrees to +45 degrees." Bitten [14] compares the agility metric definitions of several sources: Messerschmitt-Boelkow-Blohm(MBB), Eidetics, Air Force Flight Test Center(AFFTC) and General Dynamics (GD). Herbst [15] defines agility as "the derivative of the maneuver vector" and derives the agility vector from the second derivative of the velocity vector as defined in the Frenet-Serret system, [16-17]. The agility vector has the three components: (1) Longitudinal agility, in the direction of the velocity, (2) Curvature agility, in the direction of the maneuver plane and (3) Torsional agility, the rotation of the maneuver plane about the velocity vector. The agility metric for each is defined as the peak measured value versus time for a given maneuver. Eidetics [18] separates agility into three components representing (1) acceleration/deceleration along the flight path, (2) symmetrical turning perpendicular to the flight path and (3) rolling about the velocity to re-orient the flight path. The AFFTC [19] has defined two types of agility known as functional and transient agility. Transient agility is associated with maximum angular accelerations and functional agility is associated with maximum angular rates. Finally, GD [5] has defined agility as the ability to point the aircraft quickly, continue pointing the aircraft and accelerate quickly. The above sources have converged on the following relationship between state, maneuverability and agility. The state of interest is a three dimensional state vector composed of axial, pitch and roll components of the velocity vector in the Frenet-Serret system. These are airspeed, velocity vector direction as defined by heading and flight path angles and the lift plane orientation as defined by bank angle, respectively. Maneuverability is the time derivative of the state vector and is therefore related to aircraft acceleration. Agility is the time derivative of aircraft maneuverability and is therefore related to aircraft "jerk", Herbst [3].

In summary, agility can be separated into three components that quantify: (1) *Pitch/Curvature Agility* in the direction of the lift/maneuver plane, as represented by the time to capture a body axis heading or pitch angle, versus initial load factor, (2) *Lateral Roll/Torsional Agility* in the rotation of the lift/maneuver plane about the velocity vector, as represented by the time to bank versus airspeed and load factor and (3) *Longitudinal/Axial Agility* in the direction of the velocity vector, as represented by the time to capture a final airspeed versus initial airspeed and load factor. A highly agile aircraft is characterized by high sustainable g and g-onset rates, large roll rates at elevated g, large positive "specific excess power" values throughout its operating envelope and fast engine response transients. Agility is a combination of maneuverability and controllability. A highly agile aircraft consists of a highly maneuverable aircraft with substantial "jerk" that has exceptional control and response characteristics throughout its operating envelope.

The agility vector has the three components: (1) Axial agility, (2) Curvature agility and (3) Torsional agility. Herbst [15] defines the agility metric for each as the peak measured value versus time for a given maneuver. As such, the metrics are not independent. For example, a 90 degree angle of attack (α) pitch-up maneuver which is induced by high curvature agility could be used to decelerate the aircraft with high drag and therefore contribute to axial agility. Another basic maneuver utilizing curvature agility is the half-loop. Basic maneuvers usually consists of two or more metrics. For example, maneuvers utilizing curvature agility and torsional agility are, among others, the split-S, lateral turns and loaded roll reversals. Each consists of capturing desired bank angles and angles of attack.

1.3 Synthesizing Agility for Optimal Maneuvering

Herbst's peak value agility metrics provide a quantitative measure of an aircraft's potential to survive and to win engagements. Different aircraft will most surely have different agility vectors. Which is superior in surviving and in winning? That will depend on the optimal synthesis of each aircraft's agility vector. The actual time duration of a maneuver or flight path is the double integration over time of the agility vector. How do we optimize an aircraft's agility potential in performing maneuvers to survive and to win engagements? Looking back at the various definitions of agility we find that there is an underlying notion central to each of the authors and this is designing an aircraft and its control to perform tightly, quickly, rapidly, etc in maneuvering the aircraft. They refer to optimal performance of aircraft motion in some sense. For example, the agility of an aircraft could be utilized in such a manner so that the aircraft performs a desired maneuver in minimum time with a constraint on the loss of energy. An essential question is then: How do we synthesis an aircraft's agility into an overall measure of its capability to survive and to win engagement? A natural measure of such performance is the minimum time duration to perform a desired maneuver under a constraint on the loss of energy. How do we compute this minimum time? What does the optimal trajectory look like in space? The problem of optimizing such flight is a problem for optimal control theory, [20]. It can be used to synthesis an aircraft's agility into optimal trajectories. It can be used to established the form of optimal high alpha flight trajectories. It can confirm classical tactics and strategies, establish new ones and yield any improvements stemming from high alpha flight using pitch and yaw thrust vectoring (which is a key to the penetration into deep alpha space).

1.4 Previous Work on Flight Path Optimization

Performance improvements of poststall capability of a future tactical fighter were investigated in the late 70's and early 80's by Well, et al [21-23]. They use point mass equations. Control variables are angle-of-attack, bank angle, throttle and speed brake angle. Optimal control time histories are presented for aircraft with and without poststall capability. Minimum time changes are considered for both flight-path heading and fuselage pointing. Maneuvers considered are turning maneuvers such as half-loop, split-S and level turns; pointing maneuvers; slicing maneuvers and evasive maneuvers. They found a simplifying principle that governs their optimal control solutions. It is that of flying at maximum instantaneous turn rates as long as requirements on final velocity do not correspond to smaller angles of attack. For sufficiently large initial velocities near 0.9 Mach, they found that power setting and speed brakes are used so that optimal flight occurs near the corner velocity as much as possible because instantaneous turn rates outside the poststall region are the highest there; they found the optimal maneuver tending toward a half-loop maneuver. The gravitational force on the half-loop assist the deceleration process necessary in order to fly near the corner velocity. Deceleration into the poststall region, where instantaneous turn rates become very large, and subsequent acceleration to the required final velocity was observed for one turning maneuver (fixed final state) for sufficiently small initial velocities near 0.3 Mach; they found the optimal maneuver to be a split-S maneuver. For the slicing maneuver, which is a typical poststall maneuver, poststall has time advantages because of the extremely large turn rates at small velocity. In addition to time advantages of poststall flight they found other advantages such as the pointing capability and the capability of maneuvering in a small area.

Other works using the point mass equations of motion in flight path optimization are the following. Hedrick and Bryson [24] use energy and sideslip as their state variables and bank angle, throttle and Mach number as their control variables. Humphreys, et al [26]

uses Mach number, flight path angle and heading as their state variables and angle-of-attack, bank angle, and throttle as their control variables. Other authors conducting research at the Air Force Institute of Technology [27-29] use the same states and the same controls with the addition of thrust vectoring. Forsling, et al [30] uses more complex model dynamics in which he uses the point mass equations together with first order short period dynamics approximation; they also use angle-of-attack as a control variable.

Multiple time scale analysis (singular perturbations) has been applied to flight path optimizations: Kelley [31-33], Calise and Moerder [34] and Shinar, Farber and Negrin [35]. They also use point mass equations. Their control variables are angle-of-attack, bank angle and throttle.

In summary, the work of Herbst [3] and Well [21] have established potential benefits of high alpha flight for close air combat effectiveness. Very small radii of turn can be achieved by flight at very high angles-of-attack, e.g., 70-90 degrees. The key to increased performance is thrust vectoring in pitch and yaw. It will enhance agility in critical flight conditions and also enhance the decoupling of fuselage aiming and flight path control. *But, for the most part, flight path optimization research has been restricted to point mass equations of motions.* This restriction implies certain underlying assumptions. The moment equations are neglected. Angular rate contributions to the forces are neglected. Unsteady effects are neglected. States are used as control variables. Moments induced by thrust vectoring are neglected. Eliminating the moment equations leads to incorrect results, especially in post-stall flight. Thrust vectoring creates a moment which must be counter-balanced by an aerodynamic moment which in turn degrades the forces. In high alpha flight the forces are significant functions of the roll, pitch and yaw rates. To be sure, the problem of flight path optimization is more complex when moment equations are included in the optimization analysis along with the force equations. Since poststall benefits are at low speeds and high alpha, in the presence of unsteady flow and with thrust vectoring it is becoming more difficult to justify the neglect of moment equations in optimal control analysis of poststall aircraft flight.

1.5 Previous Related Work of the Principal Investigator

The principal investigator of this grant initiated research work in 1988 to conduct optimal control analysis of poststall aircraft flight in which moment equations are included in the equations of motion. Having available only a longitudinal high alpha model of the T-2C airplane, a singular perturbation outer layer analysis was initiated on the T-2C for capturing very large pitch angles in minimum time, [36]. Next, optimal control theory was applied to the same high alpha dynamics; it yielded a poststall solution, [37], which gave superior performance over that of the previous singular perturbation approach. Afterwards, under the grant NAG-1-873, the NASA Langley Research Center provided high alpha six degrees of freedom (6 DOF) wind-tunnel data on a twin tail, high performance airplane, [38], which is based on the wind-tunnel in [39-40]. In a Master's thesis Garrett, [41], and Garrett, et al, [42], investigated time optimal half-loop maneuvers for the high alpha research vehicle (HARV) model of [38]. An outer layer singular perturbation feedback control law was derived for HARV to perform the half-loop maneuver. Using Garrett's nonlinear feedback control law, the half-loop maneuver was simulated at an initial 0.6 Mach number and 15,000 ft altitude; HARV performed the half-loop in 13.12 seconds and only gained 2,500 ft altitude. The work in [36,37,41-42] considered complex longitudinal dynamics consisting of the pitch moment equation as well as the two force equations. Additional optimal control analysis of high alpha flight is investigated under the current NASA LaRC grant NAG-1-959.

1.6 Brief Statement of Grant Objectives

The main objective of the work on this grant is to synthesis an aircraft's agility into a time optimal trajectory in the performance of a prescribed maneuver. We seek to determine the shapes and forms in space of optimal high alpha flight trajectories, confirming classical tactics and strategies, establishing new ones and yielding any improvements stemming from high alpha flight, thrust vectoring, etc. The optimal control solutions for minimum time maneuvers are based on both moment and force equations. Specifically, time-optimal flight paths are to be considered for the half-loop maneuver and a 90 degree pitch-up maneuver with and without thrust vectoring. Finally, level turns are to be initially investigated using outer layer solutions of singular perturbation analysis.

Another objective of this work is to establish an analytical six degrees of freedom (6 DOF) aerodynamic model of a high angle-of-attack (alpha) combat airplane that can be utilized in optimization and control analysis/synthesis studies. Emphasis is placed on deriving such a model with validity in the altitude-Mach flight envelope centered at an altitude $h = 15,000$ feet and a Mach number $M = 0.6$. An effort is to be made to extend the validity from 0.3 to 0.9 Mach. The analytical models of aerodynamic derivatives are to be derived as nonlinear functions of alpha with all other states and control variables fixed. Interpolation is to be required between the parameterized nonlinear functions.

2. Grant Research Results on Analytical Models

Several levels of analytical aerodynamic models are derived from wind-tunnel data for subsonic high alpha flight. The first described in Section 2.1 is the most comprehensive. It requires interpolation between various nonlinear functions of alpha. Simpler analytical models are described in Sections 2.2 and 2.3 which are representative of high alpha flight but contain little or no interpolation and are, consequently, more restrictive in their potential applicability.

2.1 Analytical 6 DOF High Alpha Aerodynamic Model

The motion of aircraft in combat flight is six degrees-of-freedom consisting of both force and moment equations. The equations of motion are nonlinear, especially post-stall flight. In order to reduce complexity, most authors use point mass equations consisting only of the force equations in optimal control analysis. In our flight path optimization work we have elected to use both force and moment equations in our work. We developed our 6 DOF equations of motion following Etkin [43]. A 6 DOF analytical aerodynamic model of a high alpha research vehicle is derived and is published as a contractor's report, Cao and Stalford [44]. We derived our aerodynamic derivatives from the wind-tunnel model contained in Buttrill, et al [38] which is based on that in [39-40]. The derivation is based on wind-tunnel model data valid in the altitude-Mach flight envelope centered at 15,000 ft altitude and 0.6 Mach number with Mach range between 0.3 and 0.9. The analytical models of the aerodynamics coefficients are nonlinear functions of alpha with all control variable and other states fixed. Interpolation is required between the parameterized nonlinear functions. The lift and pitching moment coefficients have unsteady flow parts due to the time rate of change of angle-of-attack ($\dot{\alpha}$). Our initial effort in deriving an analytical aerodynamic model for the lateral mode which involved Interpolation between nonlinear functions is presented in Guy,[50]. That initial effort was improved upon to obtain better approximating analytical models for the lateral mode, [44].

The analytical models are plotted in [44] and compared with their corresponding wind-tunnel data. Piloted simulated maneuvers of the wind-tunnel model are used to evaluate the analytical model. The maneuvers considered are pitch-ups, 360 degrees loaded and unloaded rolls, turn reversals, split S's and level turns. The evaluation finds that (1) the analytical model is a good representation at Mach 0.6, (2) the longitudinal part is good for the Mach range 0.3 to 0.6 and (3) the lateral part is good for Mach numbers between 0.6 and 0.9. The computer simulations show that the storage requirement of the analytical model is about one tenth that of the wind-tunnel model and it runs twice as fast.

2.2 Analytical High Alpha Longitudinal Aerodynamic Model

The tabular wind-tunnel model [38] presented numerical difficulties in the construction of Pontryagin's maximum principle solutions for the half-loop with final times greater than 1.2 seconds. We considered using linear interpolation between the tabular data points but this would have presented the same problems as encountered in [36]. We also considered using second order splines but found that this increased the CPU time on an IBM 3090-200 far beyond a reasonable limit. Consequently, to circumvent the numerical difficulty we derived analytical model fits to the wind-tunnel aerodynamical data. We concentrated on fitting the main features of the shape of the data with less emphasis on a very close fit at all points. The aerodynamic coefficients of lift, drag and pitching moment are modelled analytically at the Mach number 0.4 and the altitude 15,000 ft. We used the Mach number 0.4 since our half-loop maneuvers started at 0.6 Mach and ended near 0.1 Mach. The altitude starts at 15,000 ft and increases to about 17,500 ft during our half-loop maneuvers. A derived analytical model is presented in Stalford, et al [45]; it requires interpolation between extreme values of the stabilator control variable. An additional analytical model is derived by Hoffman which requires no interpolation and is presented in Hoffman, et al [48]. Both models are simpler than that contained in [44].

2.3 Analytical High Alpha Lateral Aerodynamic Model

An analytical high alpha aerodynamic model for the lateral mode is derived. It is used in investigating optimal levels turns. This lateral model is representative of high alpha lateral aerodynamics contained in the wind-tunnel model,[38]. The initial effort in deriving an analytical aerodynamic model for the lateral mode which involved Interpolation between nonlinear functions is presented in Guy,[50]. Using the work of [50], Hoffman derived an analytical model for the lateral mode which requires no interpolation. That model is presented in Hoffman, et al [48]. That lateral model is simpler than that contained in [44].

3. Grant Research Results on Flight Path Optimization

Maximum principle solutions of the half-loop are describe in Section 3.1. The results of minimizing the time to perform the 90 degree pitch-up maneuver with and without thrust vectoring are described in Section 3.2. Level turns results are presented in Section 3.3.

3.1 Half-Loop Maneuver

The maximum principle approach was used to generate a candidate solution for a time-optimal half-loop maneuver of high alpha flight; the analytical longitudinal model used

in the analysis is that contained in [45] which was derived from the wind-tunnel model [38]. The work is published in [45-46]. The longitudinal aerodynamic coefficients of the wind-tunnel model [38] were analytically modelled at the Mach number 0.4 and the altitude 15,000 ft. Using the initial conditions 0.6 Mach number and 15,000 ft altitude we applied Pontryagin's maximum principle in a two point boundary value algorithm [51-52] to derive a candidate optimal solution. This solution performed the half-loop maneuver in 13.6 seconds and reached 70 degrees angle of attack at one point in the maneuver. It has a singular arc during the first third of the maneuver and a ferris wheel type solution during the last third. The singular arc occurs near maximum lift conditions and agrees with the results obtained in our singular perturbation analysis of a previous study. As a result of the attained 70 degrees angle of attack, the ferris wheel part has the nose of the airplane pointing at the target four seconds before the half-loop maneuver is completed. The importance of this work is in its comparison with a simple nonlinear feedback control law obtained previously in our singular perturbation analysis, [41-42]. The simple feedback law performs the half-loop maneuver in 13.12 seconds. The half second difference in maneuver times is accounted for by the difference in models. The analytical model derived at Mach 0.4 for this analysis has less lift than the wind-tunnel model at 0.6 Mach. The results of this work places greater significance on the simple feedback control law of singular perturbation analysis which holds for any Mach number and altitude. The effect of thrust vectoring on performing the half-loop maneuver was also analyzed; it had minimum effect on reducing the maneuver time.

3.2 Ninety (90) Degrees Pitch-Up Maneuver

We used Pontryagin's maximum principle in a two-point boundary value numerical algorithm [51-52] to derive candidate optimal open-loop controls for performing a 90 degree pitch-up maneuver in minimum time for various parametrized limits on thrust vectoring angles. The analytical longitudinal model used in the analysis is that contained in [45] which was derived from the wind-tunnel model [38]. The analysis considered three control means: stabilator, throttle setting and thrust vectoring. Without Thrust vectoring, our analytical model at initial conditions 0.6 Mach number and 15,000 feet performed the 90 degrees pitch-up in 9 seconds with the maximum throttle setting and in 7 seconds with the addition of throttle setting control. Also, using stabilator control at maximum thrust without thrust vectoring we found that the pitch-up to 74 degrees alpha took only 1.7 seconds and that there was an energy barrier beyond the alpha of 74 degrees. The additional 16 degrees required over 7 seconds of pitch-up time. Using thrust vectoring control we found that the analytical model could be pitched-up to 90 degrees alpha in only 1.5 seconds by using a maximum thrust vectoring angle of 20 degrees. For this latter case the change in altitude is less than 100 feet. Consequently by means of thrust vectoring control the 90 degrees pitch-up maneuver can be executed in a small space over a short time interval. The results are published in [47].

3.3 Level Turn Maneuver

We initiated a study to optimize turn rates and radii of high alpha flight in the horizontal plane. We first designed a preliminary analytical model of the lateral modes of the wind-tunnel model [38] for fixed 0.6 Mach and 15,000 feet altitude. Sustained values were computed at two different altitudes 15,000 ft and 10,000 ft. and were compared to values obtained previously by NASA using the wind-tunnel model. The sustained turn rates and turn radii obtained using our analytical model compared well with those obtained with the wind-tunnel model. The results are documented in [50]. Next, we used the wind-tunnel model [38] to conduct high alpha level turn analysis using outer layer solutions of singular perturbation analysis. We maximized the instantaneous horizontal plane turning

rate as a function of Mach Number. Two locally maximizing solutions were obtained. One was near 40 degrees alpha and the other near 45 degrees alpha which is poststall. The outer layer solution yielded 28 degrees per second turn rate at 0.9 Mach which decreases almost linearly to 7.2 degrees per second turn rate at 0.25 Mach. Invoking a pilot's load factor constraint of 8.5 gees limits the potential benefit of the outer layer solution; it cuts off the outer layer solution at 0.65 Mach. At 0.65 Mach the outer layer solution has a turn rate of 18 degrees per second. The classical sustained turn rate solution is 10 degrees per second and occurs at 0.58 Mach. The outer layer solution drops below the classical value at 0.3 Mach. The potential benefit of the outer layer is in the higher turn rates that it offers above 0.3 Mach. A disadvantage of the outer layer is that it is nonsustainable in Mach number; That is, as an aircraft flies the outer layer solution in level turns its Mach number decreases rather quickly. As a consequence, only turns up to a 100 degrees or so will have a shorter time when flown via the outer layer. The classical solution provides smaller turn times for turns longer than about 120 degrees. These results were presented in [53].

The outer layer solution was also computed for turn radius. The classical sustained value is a turn radius of 2400 ft and occurs at 0.28 Mach. The outer layer solution is rather flat at a turn radius of 1800 ft across Mach numbers between 0.25 and 0.9. It increases very sharply to 3000 ft at 0.2 Mach. These results were also presented in [53].

Based on an analytical model derived from the wind-tunnel data, the turn rates and turn radii in [48] compare very closely with those in [53].

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